maximum deflexion. Thus, increased range of use of the instrument is introduced without loss of sensitivity as in the previous circuit, since these series resistances can be as low as required. This arrangement has been conveniently used for speeds of the order of 50 ft/s, both in the wind tunnel and in the field, where sensitivity with the original circuit is inadequate.

At speeds for which the new circuit was primarily designed, there would be no cooling by natural convection and it is doubtful if the method used in the original circuit for allowing for day-to-day variations would have been of any great value. Furthermore, the galvanometer deflexion is a measure of the e.m.f. generated by the thermocouple, but to a different scale depending on the pre-arranged speed range selected and a calibration curve in the form of deflexion against $A\frac{\rho}{\rho_0}V$ is applicable to all ambient conditions. Therefore, no variation is made in the value of the resistance R_5 and all corrections are carried out by calculation.

Values of constants used in Hilpert's law of cooling cylinders by forced convection

R	C	m
1–4	0·891	0·330
4–40	0·821	0·385
40–4000	0·615	0·466
4000–40 000	0·174	0·618
40 000–400 000	0·024	0·805

Magnetic anisotropy measurement with an oscillation magnetometer

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A method of using an oscillation magnetometer to determine magnetic anisotropy factors in the plane of a thin disk-shaped sample lying in the $X-Z$-plane is described. Both the saturation magnetization, M_0, and the factor $(N_x - N_z)M_0$ may be obtained independently; N_x, N_y and N_z are demagnetization factors which include all contributions to magnetic anisotropy such as those arising from stress, magnetocrystalline anisotropy and shape. These results are correlated with previous work on a somewhat different type of oscillation magnetometer used to determine $(N_y - N_z)M_0$, and it is shown that a correction must be applied for both types of magnetometer when the oscillation amplitude exceeds a degree or two. Finally, it is shown that the oscillation magnetometer can be employed to determine the details of any angular dependence of magnetic anisotropy in the plane of the sample if such anisotropy is sufficiently large. These measurement techniques are particularly applicable to the determination of any dependence of M_0 on film thickness in thin ferromagnetic evaporated films, and to the determination of magnetic anisotropy in thin films evaporated and/or annealed in a magnetic field.

Some time ago an oscillation magnetometer was described by Griffiths and the author[1, 2] which allowed the factors M_0 and $(N_y - N_z)M_0$ to be obtained for an ellipsoidal ferromagnetic disk or thin film. Here M_0 is the saturation magnetization of the ferromagnetic material and N_y and N_z are demagnetization factors in the Y and Z directions, respectively. In operation, a disk-shaped specimen is supported in a saturating, homogeneous magnetic field in such a way that it can rotate about its X-axis and with the plane of the disk, which contains both X- and Z-axes, parallel to the magnetic field direction at equilibrium. When equilibrium is very slightly disturbed, the specimen will oscillate about its equilibrium position with a period (neglecting damping) of $2\pi[J/\mathscr{T}]^{\frac{1}{2}}$, where J is the moment of inertia of the disk and its container about the X-axis and $\mathscr{T}$ is the restoring torque per unit angular displacement. In general, the period will depend on disk volume, saturation magnetization, magnetic anisotropy and the applied static field H_0.

The terms $(N_y - N_z)M_0$ and $(N_x - N_z)M_0$ appear in ferromagnetic resonance experiments and are composed of possible contributions from shape anisotropy, magnetocrystalline anisotropy in single-crystal samples and stress anisotropy.[3] Without independent knowledge of these factors, ferromagnetic-resonance g values cannot be obtained. By making both ferromagnetic resonance and oscillation magnetometer measurements on the same sample, the author showed that g and M_0 were independent of the thickness of evaporated nickel films down to the smallest thickness measured, 870 Å, and that such evaporated films were usually considerably stressed.[1, 4, 5]

When the g value and saturation magnetization of a material are known, the factors $(N_y - N_z)M_0$ and $(N_x - N_z)M_0$ may be experimentally determined from resonance measurements alone. There is currently a great deal of interest, however, in thin alloy films for which g and M_0 may not be accurately known, especially in regard to possible dependence of M_0 on film thickness for very thin films[6] for use in fast switching and computer storage applications.[7, 8] The combination of resonance and oscillation magnetometer measurements on such films should be a powerful means of investigating their magnetic properties, since the oscillation magnetometer may be used to determine both $(N_y - N_z)M_0$ and $(N_x - N_z)M_0$.

The oscillation magnetometer is not generally as accurate as a good torque magnetometer,[9] nor can it measure the angular dependence of anisotropy as accurately as can a torque magnetometer or ferromagnetic resonance experiments. However, it is considerably simpler, cheaper and easier to set up and use than either of the other methods, and it yields useful quantities of direct experimental interest. In many cases, oscillation magnetometer measurements on thin films evaporated and/or annealed in a magnetic field[10] to enhance their switching properties are quite sufficient to give all needed information on the isotropic and anisotropic magnetic properties of the film. Whenever there is a variation in magnetic properties through the thickness of a disk sample, magnetometer and resonance measurements may yield somewhat different values for $(N_y - N_z)M_0$, $(N_x - N_z)M_0$ and M_0. The magnetometer yields values for the above quantities averaged through the entire thickness. On the other hand, if the film thickness is greater than a resonance skin depth or so, resonance results pertain only to an exponentially weighted part of the film within approximately a skin depth of the side being measured. Thus, the combination of oscillation magnetometer measurements and resonance measurements made separately for each surface of a film or disk can yield information concerning variation of magnetic properties in the thickness dimension. In the next section, it is shown how an oscillation magnetometer can be used to obtain $(N_x - N_z)M_0$ and M_0; then, in the final section some of the anisotropy factors which can be determined by oscillation magnetometers are tabulated.

DETERMINATION OF PLANAR ANISOTROPY

In the usual operation of an oscillation magnetometer, the rotation axis is the X-axis, taken along a diameter of the disk-shaped sample, and $(N_y - N_z)M_0$ and M_0 may be determined. This type of operation is termed the X-mode. There exists another type of operation, the Y-mode, however, which can yield information about anisotropy lying entirely in the plane of the disk. The disk again lies in a magnetic field with its plane parallel to the field direction, but rotation is now about the Y-axis passing through the centre of the disk. As shown below, the Y-mode of oscillation allows $(N_x - N_z)M_0$ and M_0 to be determined. If there is no planar anisotropy, any rotational position of the disk about the Y-axis will be a position of equilibrium and no oscillation will occur when the disk is slightly rotated. However, when planar anisotropy is present, there will exist one or more stable equilibrium positions and the disk and its container can be set into oscillation about such a position.

A scheme somewhat similar to the Y-mode of operation of an oscillation magnetometer has been suggested by Dahl and Pfaffenberger,[11] but was not used by them to obtain quantitative measurements of anisotropy. In the absence of damping, the period of oscillation for infinitesimal amplitude is given by the expression already presented. Damping arises from several sources. By using long thin Nylon threads for support, the damping and restoring torque from this source can be made negligible. Also air damping can be minimized by making the sample support rotationally symmetric. Finally, eddy-current damping is negligible at the frequencies of oscillation used, and rotational hysteresis[12] is zero as long as the magnetization of the specimen is saturated. It will, therefore, be a good approximation to neglect damping effects.

The calculation of Y-mode torque with damping neglected is carried out in the Appendix yielding, for a volume V,

$$L \simeq \frac{V(N_x - N_z)M_0^2 H_0}{H_0 + (N_x - N_z)M_0}\left(\frac{\sin 2\theta}{2}\right) \equiv \mathscr{T}_0\left(\frac{\sin 2\theta}{2}\right) \qquad (1)$$

where θ is the angle in the $X - Z$-plane, in which the specimen lies, between the Z-axis, fixed in the disk, and H_0, the applied field. This result is a good approximation as long as H_0 is appreciably larger than the anisotropy field $(N_x - N_z)M_0$ and the oscillation amplitude is of the order of 20° or less. To this approximation the system satisfies a simple pendulum equation in 2θ and the period of oscillation (for non-infinitesimal excursions) is[13]

$$\tau = 4K(k)\sqrt{(J/\mathscr{T}_0)} \qquad (2)$$

where $K(k)$ is the complete elliptic integral of the first kind, $k = \sin\theta_0$, and θ_0 is the amplitude of oscillation. For infinitesimal amplitude equation (2) becomes $\tau_0 = 2\pi\sqrt{(J/\mathscr{T}_0)}$.

Equation (2) can be rewritten in the useful form

$$\left[\frac{M_0 V}{16K^2(k)J}\right]\tau_e^2 = [(N_x - N_z)M_0]^{-1} + H_0^{-1} \qquad (2a)$$

where $\tau_e(\theta_0)$ is the experimentally determined period. Since J, V, τ_e, θ_0 and H_0 will be known in an experiment, this linear relationship between τ_e^2 and H_0^{-1} shows that both $(N_x - N_z)M_0$ and M_0 can be independently determined by plotting τ_e^2 *versus* H_0^{-1} for H_0 values sufficiently large for saturation to be maintained and for equation (1) to be a good approximation. Alternatively, if either of the above quantities is known, the other can be calculated from a single measurement of τ_e. For highest accuracy, it is still desirable, however, to measure τ_e up to as high fields as practical and extrapolate the resulting τ_e^2 dependence on H_0^{-1} to infinite field strength. If we further extrapolate the straight line obtained for high field strengths to the point where it cuts the negative H_0^{-1} axis, the quantity $(N_x - N_y)M_0$ is obtained directly from the H_0^{-1} axis intercept without any need to know the values of V, J and θ_0, provided θ_0 is the same for each τ_e measurement. It is of interest that for non-infinitesimal oscillations the time-average value of H_z is less than H_0, although the time average of H_x over an oscillation period is zero. The time dependence of θ is of the form $\theta(t) = \theta_0 \cos^{-1}\left\{dn[\sqrt{(\mathscr{T}_0/J)}t]\right\}$, where dn is a Jacobian elliptic function. When $\cos\theta$ is averaged over a period, given by equation (2), we obtain $\langle \cos\theta \rangle = \pi/2K(k) = 1 - (k/2)^2 \ldots$ Thus, $\langle H_z \rangle = \pi H_0/2K(k)$.

Equation (2a) is of the form previously obtained[1, 2] for the X-mode oscillation magnetometer with $4\pi^2$ here replaced by $16K^2(k)$ and $(N_y - N_z)M_0$ replaced by $(N_x - N_z)M_0$. Exclusive of stress and magnetocrystalline anisotropy differences between the latter two quantities, there is an important difference arising from shape effects for a circular disk-like sample. For a sample approximated as a spheroid very oblate in the Y-dimension, the shape contributions to the demagnetization factors are[3] $(N_x^s - N_z^s)M_0 = 0$ and $(N_y^s - N_z^s)M_0 \simeq (4\pi - 3\epsilon\pi^2)M_0$, where ϵ is the ratio of disk thickness to disk diameter. Thus, one measures $4\pi M_0$ plus small shape, magnetocrystalline anisotropy and stress terms in the X-mode, but only the latter terms in the Y-mode. Therefore, the Y-mode yields a more sensitive measure of anisotropy than does the X-mode, but the Y-mode period of oscillation is correspondingly greater.

The above difference may be illustrated by a comparison for nickel films between the X-mode period for $(N_y - N_z)M_0 = 4\pi M_0 \simeq 6100$ G and the Y-mode period with $(N_x - N_z)M_0 = 10$ G, produced by uniaxial stress or magnetocrystalline anisotropy. Consider a disk 1·6 cm in diameter and 0·1 μ thick and a container having a moment of inertia of 1 g cm^2 in either the X- or Y-mode. This value of J is near the minimum value obtainable with convenient sizes using container material with a density near unity. The X-mode period is 1·14 s, while that for the Y-mode is 19·9 s. An applied field H_0 of 6000 oersteds was used for these calculations. A period of 20 s is too long for both convenience and accurate measurement, but it can be reduced by making the container of low-density material such as foamed polystyrene which has a density of only about 0·03. With a moment of inertia of only 0·03, the above Y-mode period is reduced to 3·45 s, a practical value. A reduction of this order of magnitude has been observed experimentally using a foamed polystyrene sample holder. An accurate and convenient way to determine τ_e is to apply a low-frequency external torque coupled magnetically or electrically to the specimen or its holder, then vary the applied frequency until resonant oscillation is noted, using optical-lever magnification of the motion.

These results indicate that anisotropy contributions of as little as 10 G in films $0 \cdot 1\ \mu$ thick or less can be detected in the Y-mode of operation. It should be noted, however, that equation (2) cannot be used to obtain both M_0 and $(N_x - N_z)M_0$ accurately and independently when $(N_x - N_z)M_0$ is much less than the saturation field strength of the material. Since the equation applies only for static field strengths greater than that required for saturation, within the applicable range H_0^{-1} will be almost negligible compared to $[(N_x - N_z)M_0]^{-1}$ for the above case. Nevertheless, if either M_0 or $(N_x - N_z)M_0$ is known, the other can still be determined from equation (2). Such determination, in conjunction with ferromagnetic resonance measurements,[10] should allow both quantities to be separately and unambiguously determined.

ANISOTROPY FACTORS

The table summarizes some of the relations between experimentally determined $(N_x - N_z)M_0$ and $(N_y - N_z)M_0$ factors and possible physical causes of anisotropy. It has been formed using results of a previous paper where the pertinent relations are derived for ferromagnetic resonance experiments.[3] In the table, stress results are given for polycrystalline material only and involve the isotropic magnetostriction constant λ. The results of Ref. 3 may be used when isotropic magnetostriction is not a good assumption. The stress T_0 is isotropic in the plane of the sample, while T is a directed stress lying in this plane parallel to the Z-axis. The stresses are positive for tension and negative for compression. The angle θ is measured in the plane from the Z-axis, and the stable equilibrium values of θ for a circular disk-shaped sample in the Y-mode are shown in column three of the table. Note that when $\lambda T < 0$, the Z anisotropy axis is a hard rather than easy magnetization axis, and equilibrium is then found at right angles to this axis. The internal fields $3\lambda T/M_0$ or $3\lambda T_0/M_0$ arising from stress in thin films can be as large as 10^3 G.[1, 4, 5] Only first order magnetocrystalline anisotropy constants K_1' and K_1 appear in the table for $(N_x - N_z)M_0$. The substitution of these values in equation (1) is only valid when $2K_1'/M_0$ and $2K_1/M_0$ are considerably less than the applied static field H_0, as pointed out in the Appendix. Further, the absence of the second-order magnetocrystalline anisotropy constants K_2' and K_2 in $(N_x - N_z)M_0$ is contingent upon either measurements with infinitesimal oscillation amplitude or the additional conditions $K_2' \ll K_1'$, $K_2 \ll K_1$.

Except in the limit of high field strengths, the introduction of the N_x, N_y and N_z general demagnetization constants to describe second-order uniaxial magnetocrystalline anisotropy or first- and second-order cubic magnetocrystalline anisotropy is an approximation. For example, the effective internal field arising from uniaxial magnetocrystalline anisotropy with the anisotropy axis along the Z-axis may be written[3]

$$ {}^{i}\boldsymbol{H} = {}^{i}H_z \boldsymbol{i}_z = \left\{(2K_1'/M_0) + (4K_2'/M_0)[1 + (M_z/M_0)^2]\right\}(M_z/M_0)\boldsymbol{i}_z $$

where $\boldsymbol{i}_z$ is a unit vector along the Z-axis. Since the second-order term involves $(M_z/M_0)^3$, ${}^{i}H_z$ cannot, in general, be set equal to $-N_zM_z$. If one attempts to calculate the torque for the oscillation magnetometer in the manner of the Appendix, but using the above expression for ${}^{i}H_z$ instead of $-N_zM_z$, one is led to an eighth-degree equation for M_z, analogous to the fourth-degree equation otherwise present. By making measurements at sufficiently high field strengths and extrapolating to infinite field strength if necessary, these difficulties may be avoided and the results given in the table will hold. The results for cubic magnetocrystalline anisotropy apply when the [001] crystal direction coincides with the Z-axis and when the plane of the disk is the (010) crystal plane. For other directions and planes the pertinent results may be obtained from Ref. 3.

The last row in the table applies to ellipsoidal samples of dimensions a, b, c along the Z-, X- and Y-axes, respectively. The demagnetization constants have been obtained from the work of Osborn[14]; the factors K and E are complete elliptic

Anisotropy type	*Y-mode* $(N_x - N_z)M_0$	*Stable equilibrium positions,* θ	*X-mode* $[(N_y - N_z) - 4\pi]M_0$
Isotropic plane stress ($X - Z$ plane)	0	All values	$\dfrac{3\lambda T_0}{M_0}$
Directed stress (Z-axis)	$\dfrac{3\lambda T}{M_0}$	$0, \pi \quad \lambda T > 0$; $\pm\dfrac{\pi}{2} \quad \lambda T < 0$	$\dfrac{3\lambda T}{M_0}\cos^2\theta$
Uniaxial magneto-crystalline anisotropy (Z-axis)	$\dfrac{2K_1'}{M_0}$	$0, \pi \quad K_1' > 0$; $\pm\dfrac{\pi}{2} \quad K_1' < 0$	$\dfrac{2K_1'}{M_0}\cos^2\theta + \dfrac{K_2'}{M_0}\sin^2 2\theta$
Cubic magneto-crystalline anistropy	$\dfrac{2K_1}{M_0}$	$0, \pm\dfrac{\pi}{2}, \pi \quad K_1 > 0$; $\pm\dfrac{\pi}{4}, \pm\dfrac{3\pi}{4} \quad K_1 < 0$	$\dfrac{K_1}{2M_0}(3 + \cos 4\theta) + \dfrac{K_2}{2M_0}\sin^2 2\theta$
Shape anisotropy $a \geqslant b \geqslant c$	$\dfrac{4\pi c M_0}{ae^2}\left[\dfrac{(2 - e^2)E - 2(1 - e^2)K}{(1 - e^2)^{\frac{1}{2}}}\right]$	$0, \pi$	$\left(\dfrac{-4\pi c M_0}{a}\right)\left[\dfrac{E - (1 - e^2)K}{e^2(1 - e^2)^{\frac{1}{2}}}(1 + \sin^2\theta) + \dfrac{K - E}{e^2}(1 - e^2)^{\frac{1}{2}}(1 + \cos^2\theta)\right]$

integrals of the first and second kinds, both having the argument $e = [1 - (b/a)^2]^{\frac{1}{2}}$. For a circular disk $b = a$, and these results reduce to $(N_x - N_z)M_0 = 0$ and

$$[(N_y - N_z) - 4\pi]M_0 = -3\pi^2 cM_0/a$$

For an elongated ellipsoid with b appreciably smaller than a but still much larger than c, $(N_x - N_z)M_0$ reduces to approximately $(c/b)4\pi M_0$. If this quantity is of the order of 5 or 10 G, it should be possible to measure M_0 in a thin film of this shape quite accurately using the Y-mode of operation. Generally, however, other anisotropy effects in thin films will outweigh that of shape even in an elongated sample.

For the X-mode column, the values of $[(N_y - N_z) - 4\pi]M_0$ have been listed rather than $(N_y - N_z)M_0$. The $4\pi M_0$ factor is almost always the dominant term in $(N_y - N_z)M_0$. Therefore, the values listed in the last column usually represent relatively small corrections to $4\pi M_0$, and themselves cannot be determined as accurately as the corresponding factors in the Y-mode. However, angular dependence can be determined in the X-mode but not in the Y-mode. Here again θ measures the angle in the $X - Z$ plane between the applied magnetic field direction (no X-mode oscillation) and the Z-axis, fixed in the sample. By carrying out X-mode observations with a circular sample securely fixed in various angular positions with respect to the applied field direction, the correction terms listed in the last column of the table may be determined, provided their amplitudes are of the order of $0{\cdot}01(4\pi M_0)$ or greater.

REFERENCES

(1) MACDONALD, J. R. D.Phil. Thesis (University of Oxford, 1950).

(2) GRIFFITHS, J. H. E., and MACDONALD, J. R. *J. Sci. Instrum.*, **28**, p. 56 (1951).

(3) MACDONALD, J. R. *Proc. Phys. Soc.* [*London*] *A*, **64**, p. 968 (1951).

(4) MACDONALD, J. R. *Phys. Rev.*, **81**, pp. 312*A*, 329*A* (1951).

(5) GRIFFITHS, J. H. E. *Physica*, **17**, p. 253 (1951).

(6) CRITTENDEN, E. C., JR, and HOFFMAN, R. W. *Rev. Mod. Phys.*, **25**, p. 310 (1953).

(7) CONGER, R. L., and ESSIG, F. C. *Phys. Rev.*, **104**, p. 915 (1956).

(8) SMITH, D. O. *Phys. Rev.*, **104**, p. 1280 (1956).

(9) WILLIAMS, H. J. *Rev. Sci. Instrum.*, **8**, p. 56 (1937).

(10) MACDONALD, J. R. *Phys. Rev.*, **106**, p. 890 (1957).

(11) DAHL, O., and PFAFFENBERGER, J. *Z. Phys.*, **71**, p. 93 (1931).

(12) BRAILSFORD, F. *J. Instn Elect. Engrs*, **83**, p. 566 (1938).

(13) WOODS, F. S. *Advanced Calculus*, p. 370 (Boston: Ginn and Co., 1934).

(14) OSBORN, J. A. *Phys. Rev.*, **67**, p. 351 (1945).

(15) LANDAU, L., and LIFSHITZ, E. *Phys. Z. Sowjet.*, **8**, p. 153 (1935).

APPENDIX

Torque calculation for the Y-mode

The restoring torque L is given by

$$L = VM \times H_0 = -VM \times {}^iH \tag{3}$$

where ${}^iH$ is that part of the internal field not including the external field. The direction of the magnetization vector may be determined from the Landau–Lifshitz equation[15] which reduces very closely to

$$M \times H^i = M \times (H_0 + {}^iH) = 0 \tag{4}$$

in the present low-frequency case. Its magnitude satisfies $|M|^2 \equiv M_0^2 \simeq M_x^2 + M_z^2$, since M_y is very nearly zero. The total internal field components in the present case are $H_x^i = H_0 \sin\theta - N_x M_x$, $H_y^i \simeq 0$, $H_z^i = H_0 \cos\theta - N_z M_z$. Equations (4) now yields

$$M_x H_z^i = M_z H_x^i = M_x[H_0 \cos\theta - N_z M_z] = = M_z[H_0 \sin\theta - N_x M_x] \tag{5}$$

or

$$M_x = \frac{M_z H_0 \sin\theta}{H_0 \cos\theta + (N_x - N_z)M_z} \tag{6}$$

On using the second part of equation (3), introducing the internal field expressions, and making use of equation (6), one finds

$$L = L_y = V(N_x - N_z)M_x M_z = = \frac{V(N_x - N_z)M_z^2 H_0 \sin\theta}{H_0 \cos\theta + (N_x - N_z)M_z} \tag{7}$$

Finally, using equation (6) and the saturation condition, the equation determining M_z is found to be

$$[H_0 \cos\theta + (N_x - N_z)M_z]^2 = = \left(\frac{M_z}{M_0}\right)^2 \left\{[H_0 \cos\theta + (N_x - N_z)M_z]^2 + H_0^2 \sin^2\theta\right\} \tag{8}$$

This is a fourth-degree equation in M_z. It may be readily solved to first-order in $[(N_x - N_z)M_0/H_0]$ provided this quantity is appreciably less than unity. The result is

$$M_z \simeq M_0 \cos\theta\left[1 + \frac{(N_x - N_z)M_0}{H_0}\sin^2\theta\right] \tag{9}$$

The torque L thus becomes, approximately

$$\left.\begin{aligned} L &\simeq \frac{V(N_x - N_z)M_0^2 H_0 \cos^2\theta \sin\theta\{1 + [(N_x - N_z)M_0/H_0]\sin^2\theta\}^2}{H_0 \cos\theta + (N_x - N_z)M_0 \cos\theta\{1 + [(N_x - N_z)M_0/H_0]\sin^2\theta\}} \\ &\simeq \frac{V(N_x - N_z)M_0^2 H_0(\sin^2\theta/2)\{1 + [2(N_x - N_z)M_0/H_0]\sin^2\theta\}}{H_0 + (N_x - N_z)M_0 + [(N_x - N_z)M_0]^2 \sin^2\theta/H_0} \end{aligned}\right\} \tag{10}$$

This equation is correct to second-order in $[(N_x - N_z)M_0/H_0]$. Since, however, the terms in the numerator and denominator proportional to $[(N_x - N_z)M_0/H_0]^2$ are multiplied by sinusoidal factors which will be small for relatively small oscillation amplitude, these terms may usually be neglected. Equation (10) then reduces to equation (1) of the text.